

# A note on the consistency of Hybrid Eulerian/Lagrangian approach to multiphase flows

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## Abstract

The aim of the present paper is to introduce and to discuss inconsistencies errors that may arise when Eulerian and Lagrangian models are coupled for the simulations of turbulent poly-dispersed two-phase flows. In these hybrid models, two turbulence models are in fact implicitly used at the same time and it is essential to check that they are consistent, in spite of their apparent different formulations. This issue appears in particular in the case of very-small particles, or tracer-limit particles, and it is shown that coupling inconsistent turbulence models (Eulerian and Lagrangian) can result in non-physical results, notably for second-order fluid velocity moments. This problem is illustrated by some computations for fluid particles in a turbulent channel flow using several coupling strategies.

*Keywords:*

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## 1. Introduction

Polydispersed turbulent two-phase flows are found in numerous environmental and industrial processes, very often in contexts that involve additional issues, for example chemical and combustion ones (Clift et al. , 1978). From a practical point of view, the Navier-Stokes equations (for the fluid) and the particle equations (for the particulate phase) must be solved. Generally speaking, two basic approaches have been used for the description of the particle phase: the *Eulerian*, or *two-fluid* approach, where the dispersed particle phase is treated as a fluid in much the same way as the carrier phase, namely by a set of continuum equations which represent the conservation of statistical means, such as mass, momentum and energy, within

some elemental volume of the dispersed phase and the so-called *Lagrangian*, or *particle tracking*, approach where individual particles are tracked through the computed fluid field by solving the individual particle equation of motion. In the latter case, the complete method for the two phases constitutes a hybrid Eulerian/Lagrangian approach. It is worth remembering that various strategies have been explored to couple different Lagrangian tracking approaches with Eulerian approaches for the fluid phase, in particular, with RANS/Moments approach (Stock , 1994; Muradoglu et al. , 2001; Minier et al. , 2004), LES (Boivin et al. , 2000; Jaber et al. , 2002; Okong & Bellan , 2004), and DNS (Soldati & Marchioli , 2009; Bec et al , 2006; Toschi & Bodenshatz , 2009; Eaton , 2009).

Let us now introduce the general nature of the Lagrangian approach. A no-model approach, in the spirit of DNS, is possible, but, in practice, the exact equations of motion are not treatable in realistic cases. Indeed, in the case of a large number of particles and of turbulent flows at high Reynolds numbers, the number of degrees of freedom turns out to be huge and one has to resort to a contracted probabilistic (modeled) description. In this case, particles are represented by an ensemble of Lagrangian stochastic particles whose properties are driven by either a model given in the form of a set of stochastic differential equations (continuous SDEs) (Minier & Peirano , 2001; Chibbaro & Minier , 2008) or directly in terms of a numerical scheme (random walk) (Stock , 1994; Pope , 1987; MacInnes & Bracco , 1992). The solution of the set of stochastic equations represents a Monte Carlo simulation of the underlying pdf. Therefore, this approach is equivalent to solving directly the corresponding equation for the pdf in the corresponding sample-space. In this work, we shall use the continuous approach. It is important to underline here that this choice is made for the sake of clarity and without loss of generality, since the Langevin approach is expressed in terms of continuous variables and, thus, is physically more intuitive. The random walk models share the same properties but they are discrete and therefore they correspond to a numerical scheme for a given continuous stochastic model.

In turbulent two-phase flows, the SDEs equations of the model contain several mean fields and have the general form

$$dZ_i(t) = A_i(t, \mathbf{Z}, \langle f(\mathbf{Z}) \rangle) dt + \sum_j B_{ij}(t, \mathbf{Z}, \langle f(\mathbf{Z}) \rangle) dW_j(t), \quad (1)$$

where the operator  $\langle \rangle$  stands for the mathematical expectation. For exemple,

a typical Langevin model has the form (Minier & Peirano , 2001)

$$dx_{p,i} = U_{p,i} dt \quad (2)$$

$$dU_{p,i} = \frac{1}{\tau_p} (U_{s,i} - U_{p,i}) dt \quad (3)$$

$$dU_{s,i} = -\frac{1}{\rho_f} \frac{\partial \langle P \rangle}{\partial x_i} dt + (\langle U_{p,j} \rangle - \langle U_{f,j} \rangle) \frac{\partial \langle U_{f,i} \rangle}{\partial x_j} dt \\ - \frac{1}{T_{L,i}^*} (U_{s,i} - \langle U_{f,i} \rangle) dt + \sqrt{\langle \epsilon \rangle \left( C_0 b_i \tilde{k}/k + \frac{2}{3} (b_i \tilde{k}/k - 1) \right)} dW_i \quad (4)$$

where quantities such as  $T_{L,i}^*$ ,  $\tilde{k}$  etc. are defined precisely elsewhere (Minier & Peirano , 2001; Minier et al. , 2004). For the sake of present discussion, the important point is that this form reveals that different mean fields enter the model equations. Fluid mean fields, typically  $\langle U_{f,i} \rangle$ , are provided by the Eulerian solver while particle mean fields, such as  $\langle U_{p,i} \rangle$ , are extracted directly from the particles and are therefore provided by the Lagrangian solver. As such, in the case of particles with a non-negligible inertia, the problem is well-posed since the different mean fields come from different sources (Eulerian/Lagrangian solvers). In that situation, it may even be tempting to believe that improving the prediction of one mean field, for example the fluid mean velocity, results in improving the overall capacity of the complete model.

However, the present hybrid method raises issues of consistency between the Eulerian and the Lagrangian solvers. This issue is particularly appreciable in the tracer limit (fluid particle), for which the stochastic two-phase flow model reduces simply to a fluid model. This asymptotic limit-case represents the most relevant and fundamental situation where to test the effect of possible errors due to inconsistency, for two main reasons: (1) in this limit, as will be shown below, possible inconsistency can be seen directly at the level of fluid moments; (2) one of the main application in bounded flows concerns particle deposition, where tracer particle are precisely the most important (aerosols) and, often, the least-well predicted by standard models. In the particle tracer limit of vanishing inertia, the model (1) takes the form:

$$dx_{f,i} = U_{f,i} dt \quad (5)$$

$$dU_{f,i} = -\frac{1}{\rho} \frac{\partial \langle P \rangle}{\partial x_i} dt - \frac{1}{T_L} (U_i - \langle U_i \rangle^E) dt + \sqrt{C_0 \langle \epsilon \rangle} dW_i , \quad (6)$$

where  $T_L$  represents the Lagrangian time-scale of velocity correlations and

it is defined by  $T_L = \frac{1}{(1/2+3/4C_0)} \frac{k}{\langle \epsilon \rangle}$ . This model corresponds to the Simplified Langevin Model (SLM) (Pope , 1994b). In the above Langevin model, the fluid mean velocity field appearing in the rhs of the equation,  $\langle U_i \rangle^E$ , is provided by the Eulerian solver as indicated by the index E. However, since we are now dealing with fluid particles, the Lagrangian mean velocity field extracted directly from the particles,  $\langle U_i \rangle^L$ , represents the *same* physical property. We are therefore in presence of a *duplicate field* and the consistency issue requires that  $\langle U_i \rangle^L = \langle U_i \rangle^E$ . Yet, these two mean fields result in fact from two different sources:  $\langle U_i \rangle^E$  results from the turbulence model chosen in the Eulerian solver whereas  $\langle U_i \rangle^L$  results from the Langevin model. Thus, it is not obvious *a priori* to know what happens when the Eulerian physical model and the PDF one are not consistent. For instance, coupling Eulerian mean fields computed through DNS with a Lagrangian model which is consistent with a given RANS model (Pope , 1994b; Muradoglu et al. , 1999, 2001) may introduce a bias error. Unfortunately, this point has not yet received any attention and it has been too quickly believed that a "better" mean field  $\langle U_i \rangle^E$  fed into the simple Langevin model would automatically bring about a "better" model. This route has been already used in many works, notably in particle deposition cases (Kroger & Drossinos , 2000; Matida et al. , 2000; Tian & Ahmadi , 2007; Dehbi , 2008; Parker et al. , 2008; Zhang & Chen , 2009). Even though in some cases and with some specific models this procedure may turn out to be valid, it should be taken with care in general. This procedure is based upon the idea that the Eulerian and the Lagrangian parts of a hybrid method are completely independent, in absence of two-way coupling. It is our purpose to review critically this method.

The question addressed in the present work is: *what is the consequence on the particle simulations of using two inconsistent turbulence models (the turbulence model used in the Eulerian solver and the one corresponding to the Langevin model)?* In order to develop a quantitative example, the purpose of this work is to investigate numerically this effect in a turbulent channel flow which is taken here as a relevant engineering case.

## 2. Model consistency issue

Following the discussion in the introduction, it is worth recalling that, in terms of Eulerian mean equations, the SLM model is equivalent to the

following turbulence model (Pope , 1994a,b; Minier & Peirano , 2001):

$$\frac{\partial \langle U_i \rangle}{\partial x_i} = 0 \quad (7)$$

$$\frac{\partial \langle U_i \rangle}{\partial t} + \langle U_j \rangle \frac{\partial \langle U_i \rangle}{\partial x_j} + \frac{\partial \langle u_i u_j \rangle}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \langle P \rangle}{\partial x_i} \quad (8)$$

$$\begin{aligned} \frac{\partial \langle u_i u_j \rangle}{\partial t} + \langle U_k \rangle \frac{\partial \langle u_i u_j \rangle}{\partial x_k} + \frac{\partial \langle u_i u_j u_k \rangle}{\partial x_k} = & -\langle u_i u_k \rangle \frac{\partial \langle U_j \rangle}{\partial x_k} - \langle u_j u_k \rangle \frac{\partial \langle U_i \rangle}{\partial x_k} \\ & - \frac{2}{T_L} \langle u_i u_j \rangle + C_0 \langle \epsilon \rangle \delta_{ij}. \end{aligned} \quad (9)$$

Using the expression retained for  $T_L$  in Eq. (6), the transport equation for the second-order moments can be re-expressed as :

$$\begin{aligned} \frac{\partial \langle u_i u_j \rangle}{\partial t} + \langle U_k \rangle \frac{\partial \langle u_i u_j \rangle}{\partial x_k} + \frac{\partial \langle u_i u_j u_k \rangle}{\partial x_k} = & -\langle u_i u_k \rangle \frac{\partial \langle U_j \rangle}{\partial x_k} - \langle u_j u_k \rangle \frac{\partial \langle U_i \rangle}{\partial x_k} \\ & - \left(1 + \frac{3}{2}C_0\right) \left(\langle u_i u_j \rangle - \frac{2}{3}k\delta_{ij}\right) - \frac{2}{3}\delta_{ij}\langle \epsilon \rangle. \end{aligned} \quad (10)$$

This shows that the SLM corresponds to a  $R_{ij} - \epsilon$  Rotta model (Pope , 1994b). We have retained this simple version, namely the SLM model, which is consistent with usual Reynolds-stress models as a kind of sound basis for the numerical investigations on particle deposition though it is clear that, at least for the prediction of fluid mean quantities, this leaves room for improvement by using more complex Langevin ideas.

### 3. Numerical Results

A stationary turbulent channel-flow is solved first with the Eulerian method and, then, with the PDF one. The computations have been performed for the case of fully developed turbulent channel flow at  $Re = u_\tau h / \nu = 395$ . The channel flow DNS results of Moser *et al.* (Moser et al. , 1999) have been taken for comparison as reference data. For the simulations, the mesh used in DNS (128 points along  $y$ ) is also used here, which assures a good interpolation of the mean fields for the Lagrangian stochastic particles. Quantities designed with the upper-script + are non-dimensionalised with wall parameters. Being a particle-mesh method, particles move in a mesh where, at every cell, the mean fields describing the fluid are known. Generally

speaking, the statistics extracted from the variables attached to the particles, which are needed to compute the coefficients of system (5)-(6) are not calculated for each particle (this would cost too much CPU time) but are evaluated at each cell center following a given numerical scheme (averaging operator). The stochastic equations (5)-(6) are solved through a consistent first-order, unconditionally stable numerical scheme (Minier et al. , 2003). No-slip and impermeability conditions are satisfied by imposing boundary conditions on the stochastic particles. Symmetry conditions are imposed at the other boundary placed at the center of the channel  $y = h/2$ . Along the  $x$  direction, periodic conditions are imposed. The details of the numerical approach have been exhaustively described in a recent article (Peirano et al. , 2006).

### 3.1. Consistency

The first set of numerical experiments has been performed coupling the Eulerian Rotta Reynolds-stress model with the Langevin Simple model previously described. Identical results are expected, since the equations for the moments of first and the second order are the same for both models and, thus, consistency is assured. In figures 1a, the mean-velocity profiles obtained with the present hybrid configuration are shown. In figures 1b, the profiles of second-order moments, that is the Reynolds stress tensor, are also shown, for the present hybrid and for the Eulerian calculations. The Eulerian and Lagrangian profiles are in a quite good agreement. In conclusion, the mean fluid velocity moments derived from the present PDF method are in agreement with those computed in the Eulerian configuration and thus with the physical expected values. Moreover, the global hybrid method Eulerian (Rotta-model)/Lagrangian (SLM) is demonstrated to be consistent. It is worth emphasising that the ingredients that have been necessary to reach this objective are:

- (i) Consistent physical model.
- (ii) Consistent numerical scheme
- (iii) Accurate global numerical method , concerning also the exchange of information from Eulerian solver to Lagrangian one.

### 3.2. Hybrid Consistency error

We can now study the global error which is possibly introduced by using a hybrid method not completely consistent. Generally speaking, hybrid

Eulerian/Lagrangian methods are affected by different kind of errors due to: (1) spatial discretisation; (2) time discretisation; (3) the use of a finite number of particle and per cell (statistical and bias errors). All these errors have been made negligible in the following simulations, in order to isolate the “hybrid consistency” error. In both methods, a time-step of  $10^{-4}s$  and a spatial-step of  $10^{-4}\delta$  have been used, which have been shown in numerical simulations to be sufficiently small for our purposes.  $5 * 10^4$  particles have been employed, which can be considered high enough in the present case. For the sake of clarity, the configuration discussed in the last section, where Eulerian mean variables consistent with Lagrangian ones were used, will be called the *standard* configuration in the following.

First, let us consider the following configuration: an Eulerian DNS (Moser et al. , 1999) is now coupled to the present Lagrangian method (this means that the mean fluid velocity  $\langle U \rangle^E$  provided to the SLM model is given by a DNS calculation). In figure 2a, the mean velocity computed by the PDF method in this configuration, together with DNS original profiles as well as the mean velocity computed by PDF method in the consistent  $R_{ij}$ -PDF configuration, are shown. The mean velocity obtained in the DNS/PDF configuration is in good agreement with the DNS one and, thus, is strongly different from the one obtained in the *standard* configuration. In fact, the exact profile is now recovered. In figure 2b, the Reynolds stress are shown for the same configurations. The profiles are now dramatically changed in comparison with the *standard* results. The  $\langle uv \rangle$  and the  $\langle u^2 \rangle$  components are strongly over-predicted. On the contrary, the other diagonal components do not present appreciable changes. This behaviour can be explained. As diagonal  $\langle v^2 \rangle$  and  $\langle w^2 \rangle$  components are essentially dependent on Lagrangian time-scale and on diffusion coefficient, they are independent of the Eulerian model and therefore not affected by the consistency error. On the contrary, the cross-shear stress  $\langle uv \rangle$  depends not only on the turbulent kinetic energy and the Lagrangian time-scale but also explicitly on the gradient of the mean velocity. It is therefore much more sensitive to the prediction of the Eulerian mean velocity. The present mean velocity profile attains a higher maximum than in the *standard* configuration and, thus, it is much steeper. The shape of  $\langle u^2 \rangle$  is in turn a direct consequence of this behavior. It is important to stress here that DNS/PDF Reynolds stress profiles are unphysical; notably, while the negative peak of the cross-shear stress *should* be of the order of 1, it turns

out to be  $\langle uv \rangle \approx -3$ .

Finally, we analyse the results obtained by coupling the present PDF model to a low-Reynolds number RANS model known for his good performance in boundary flows, the v2f model (Durbin , 1991). In the present calculations, we use a refined version of the model (Laurence et al. , 2005). In figure 3a, we show the mean velocity computed in this hybrid configuration together with the DNS and *standard* PDF results. As previously noted, the mean velocity given by the PDF method is approximately equal to the Eulerian mean one provided to the PDF solver, in this case computed by v2f solver. Furthermore, this result is also in good agreement with DNS result. In figure 3b, the Reynolds stress profiles for the same configurations are shown. The results are similar to those obtained in the hybrid DNS/PDF configuration. Even though the  $\langle uv \rangle$  and the  $\langle u^2 \rangle$  components are less overpredicted, they still show qualitatively the same behaviour and remain unphysical. The consistency error is slightly less important but still large.

Some observations are in order:

- (i) In all configurations, the mean velocity computed from stochastic particles basically collapses on the value given by the Eulerian mean velocity used in the Lagrangian model. This is in line with the physics of the Langevin model, which is based on a return-to-equilibrium idea (Minier & Pozorski , 1997).
- (ii) In the PDF-DNS and PDF-V2F configurations, a large difference between the Eulerian and the Lagrangian results at the level of the second-order statistical moments (Reynolds stress) is found. This is a direct consequence of coupling, in a hybrid approach, two methods which are not consistent. Thus, the hybrid consistency errors are identified by comparing the actual Reynolds stress profiles with those computed in the *standard* configuration, see figs. 2b,3b.
- (iii) Results obtained in the hybrid DNS/PDF and v2f/PDF approaches are qualitatively similar, even though the DNS and the v2f approaches are quite different from a theoretical point of view. This should be expected. The profiles provided by the Eulerian solver to the Lagrangian one are the mean velocity, the mean pressure, the turbulent energy and the turbulent dissipation. For these variables, the v2f approach gives



results which are in good agreement with the DNS ones. Therefore, from the point of view of the hybrid method DNS and v2f approaches are qualitatively similar. Moreover, from a quantitative point of view, the results obtained through the hybrid v2f/PDF approach are in better agreement with those calculated in the *standard*  $R_{ij}$ /PDF case than those obtained using the DNS/PDF approach, fig. 3b. Therefore, v2f model is found to be more consistent with present Lagrangian model than DNS and the hybrid consistency error is smaller.

#### 4. Conclusions

A study of a turbulent channel flow has been carried out in the framework of a Hybrid Eulerian/Lagrangian approach with the main attention pointed to the issue of the consistency between the two approaches.

We have used different kind of Eulerian methods (RANS  $R_{ij}$  and v2f, DNS) coupled with the simplified Langevin model for the Lagrangian tracking of fluid particles. It is worth underlying that this kind of couplings are normally used in multiphase simulations, for instance DNS (Matida et al. , 2000; Tian & Ahmadi , 2007; Dehbi , 2008), v2f (Zhang & Chen , 2009) among others. The Lagrangian model proposed is in the form of a set of stochastic differential equations (5)-(6) and, in practice, the PDF underlying this stochastic process is obtained via a Monte Carlo method, that is through the simulation of a number of stochastic particles. This feature is very useful to recognize immediately the physical contain of the model, at variance with other more heuristic but also very popular approaches like the discrete random walk models.

We have analysed the changes produced in the Lagrangian results by choosing different Eulerian methods. In particular, we have studied the duplicated variables which are the two first moments and we have shown that the global method is found to be consistent when  $R_{ij}$  Rotta model is coupled with the SL model, as it is theoretically expected. On the contrary, results obtained in other configurations give evidence of a bias error precisely due to the inconsistency between the Eulerian and Lagrangian physical models. It is worth emphasizing here that it is possible to assess the global consistency of the method, because we use a completely consistent and an accurate numerical scheme, which allows us to consider the numerical errors as negligible.

These results underline that, in hybrid Eulerian/Lagrangian approaches to turbulent flows, delicate questions of consistency between the two parts

of the method ineluctably arise and deserve a careful treatment. In general, the simple statement that a "better fluid profile" help to improve Lagrangian results (independently of the Lagrangian model) is not true. At variance with this belief, using an Eulerian model which is not consistent with the Lagrangian one introduces a consistency error which can introduce a flaw in the global method.

Finally, in authors' opinion, the Lagrangian tracking method is too often considered as a *black box* tool scarcely important from a physical point of view and thus hierarchically subjected to the Eulerian method. In fact, we think that in hybrid Eulerian/Lagrangian approaches to two-phase flows the Lagrangian part is more fundamental and has the guiding role. In this sense, our results show that it could be wise to start from the choice of the Lagrangian model and afterwards to choose an Eulerian model which is consistent with it.

J. Bec, L. Biferale, G. Boffetta, A. Celani, M. Cencini, A. Lanotte, S. Musacchio, F. Toschi Acceleration statistics of heavy particles in turbulence *J. Fluid Mech.*, **550**, 349 (2006).

M. Boivin, O. Simonin, and K.D. Squires. On the prediction of gas-solid flows with two-way coupling using large eddy simulation. *Phys. Fluids*, 12:2080–2090, 2000.

S. Chibbaro, J.-P. Minier, *Journal of Aerosol Science*, 39 (7), p.555-571, 2008

R. Clift, J. R. Grace, and M. E. Weber. *Bubbles, Drops and Particles*. Academic Press, New York, 1978.

A. Dehbi "Turbulent particle dispersion in arbitrary wall-bounded geometries: A coupled CFD-Langevin-equation based approach", *International Journal of Multiphase Flow* , **34** (2008) 819828 .

P.A. Durbin. "Near-wall turbulence closure modeling without damping functions." *Theor. Comput. Fluid Dyn.*, **3**, 113, (1991).

J.K. Eaton Two-way coupled turbulence simulations of gas-particle flows using point-particle tracking *Int. Journal Multiphase flows*, **35**, 792, 2009.

F.A. Jaber L.Y.M. Gicquel, P. Givi and S. B. Pope. Velocity filtered density function for large eddy simulation of turbulent flows. *Physics Fluids*, 14:1196–1213, 2002.

- C. Kroger and Y. Drossinos, “A random-walk simulation of thermophoretic particle deposition in a turbulent boundary layer”, *Int. Journal Multiphase Flow*, **26**, 1325 (2000).
- Laurence D. R., Uribe J.C., and Utyuzhnikov S.V. ”A robust formulation of the v2-f model.” *Flow, Turbulence and Combustion*, **73**, (3-4):169185, (2005).
- J.M. MacInnes and F.V. Bracco, “Stochastic particle dispersion modeling and the tracer-particle limit”, *Phys. Fluids A*, **4**, 2809 (1992).
- E.A. Matida, K. Nishino and K. Torii, “Statistical simulation of particle deposition on the wall from turbulent dispersed pipe flow”, *International Journal of Heat and Fluid Flow*, **21**, 389 (2000).
- J-P. Minier and J. Pozorski. Derivation of a pdf model for turbulent flows based on principles from statistical physics. *Phys. Fluids*, 9(6):1748–1753, 1997.
- J-P. Minier and E. Peirano. The PDF approach to polydispersed turbulent two-phase flows. *Physics Reports*, 352 (2001) 1-214.
- J-P. Minier, E. Peirano and S. Chibbaro Weak first- and second-order numerical schemes for stochastic differential equations appearing in Lagrangian two-phase flow modeling. *Monte Carlo Meth. and Appl.* 9 (2003) 93-133.
- J.P. Minier, E. Peirano and S. Chibbaro, PDF model based on Langevin equation for polydispersed two-phase flows applied to a bluff-body gas-solid flow, *Phys. Fluids* **16**, 2419 (2004).
- R.D. Moser, J. Kim, and N.N. Mansour Direct numerical simulation of turbulent channel flow up to  $Re_\tau = 590$ . *Phys. Fluids*, 11 (1999) 943-945.
- M. Muradoglu, P. Jenny, S.B. Pope, and D.A. Caughey. A consistent hybrid finite volume/particle method for the pdf equations of turbulent reactive flows. *J. Comput. Phys.*, 154:342–371, 1999.
- M. Muradoglou, S.B. Pope, and D.A. Caughey The hybrid method for the PDF equations of turbulent reactive flows: consistency and correction algorithms. *J. Comput. Phys.*, 172 (2001) 841-878.

- N.A. Okong'o and J. Bellan. Consistent large-eddy simulation of a temporal mixing layer laden with evaporating drops. part 1. direct numerical simulation, formulation and *a priori* analysis. *J. Fluid Mech.*, 499:1–47, 2004.
- S. Parker, T. Foat, and S. Preston “Towards quantitative prediction of aerosol deposition from turbulent flows”, *Journal of Aerosol Science*, **39**, 99 (2008).
- E. Peirano, S. Chibbaro, J. Pozorski and J.P. Minier “Mean-Field/PDF Numerical approach for polydispersed turbulent two-phase flows”, *Progress in Energy and Combustion Sciences*, **32**, 315 (2006).
- S. B. Pope, “Consistency conditions for random-walk models of turbulent dispersion”, *Phys. Fluids*, **30**, 2374 (1987).
- S. B. Pope. Lagrangian pdf methods for turbulent reactive flows. *Ann. Rev. Fluid Mechanics*, 26:23–63, 1994.
- S. B. Pope, “On the Relationship between Stochastic Lagrangian Models of Turbulence and Second-Order Closures”, *Phys. Fluids*, **6**, 973 (1994).
- A. Soldati, and C. Marchioli Physics and modelling of turbulent particle deposition and entrainment: Review of a systematic study. *Int. Journal Multiphase flows*, **35**, 827, 2009.
- D. E. Stock. Particle dispersion in flowing gases. *J. Fluids Eng.*, 118:4–17, 1996.
- L. Tian, and G. Ahmadi “Particle deposition in turbulent duct flowscomparisons of different model predictions”, *Journal of Aerosol Science*, **38**, 377 (2007).
- F. Toschi, E. Bodenschatz Lagrangian Properties of Particles in Turbulence. *Ann. Review of Fluid Mech.*, 41, 375 (2009) .
- W.C. Welton. Two-dimensional PDF/SPH simulations of compressible turbulent flows. *J. Comput. Phys.*, 139 (1998) 410-443.
- Z. Zhang, and Q. Chen “Prediction of particle deposition onto indoor surfaces by CFD with a modified Lagrangian method”, *Atmospheric Environment*, **43** (2009) 319328.

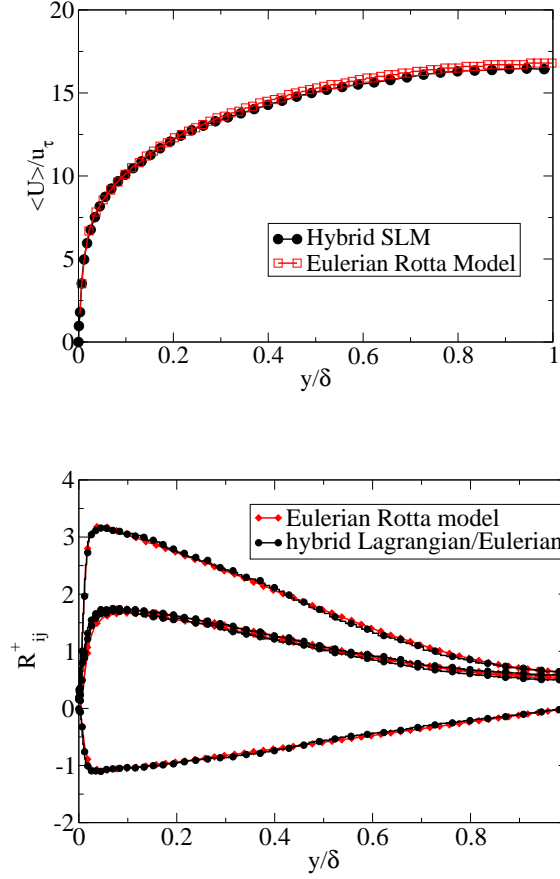


Figure 1: Mean velocity (a) and Reynolds stress (b) results obtained in the Eulerian method, in the present Hybrid configuration and in the DNS (Moser et al. , 1999). The results are in non-dimensional units.

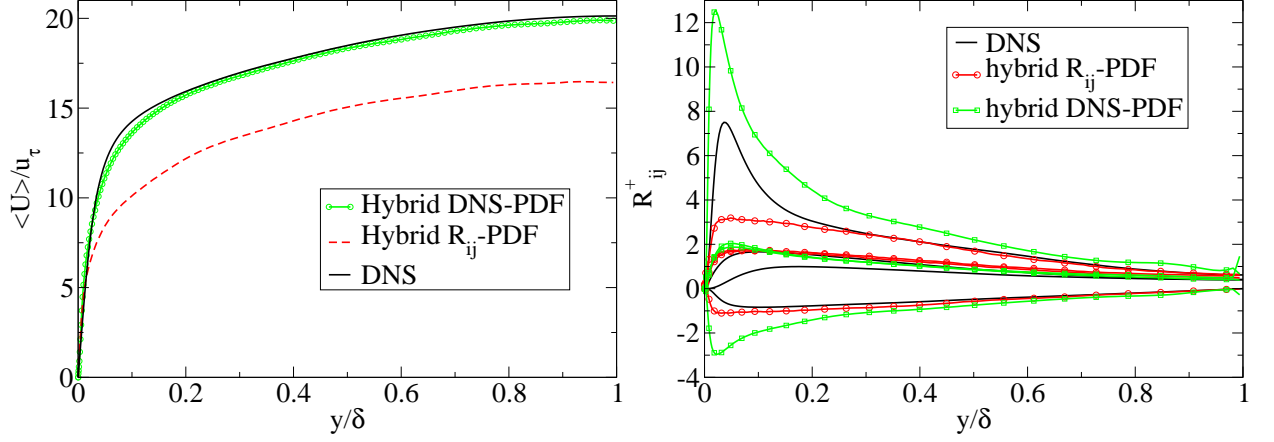


Figure 2: DNS-PDF configuration: (a) mean velocity. (b) Reynolds stress. DNS (Moser et al. , 1999) and  $R_{ij}$ /PDF profiles are also give for comparison.

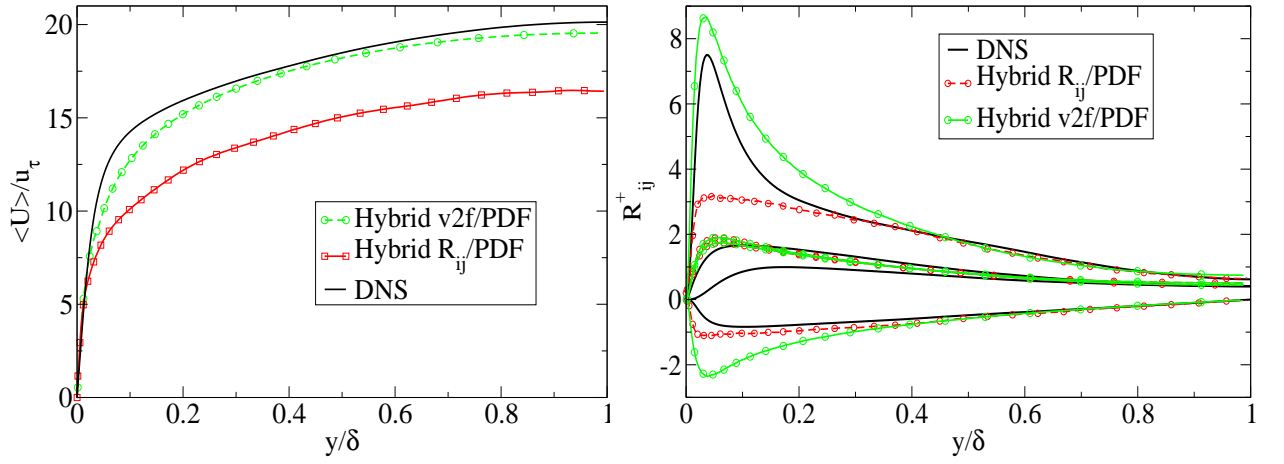


Figure 3: v2f-PDF configuration: (a) mean velocity. (b) Reynolds stress. DNS (Moser et al. , 1999) and  $R_{ij}$ /PDF profiles are also give for comparison.